Dynamic thermal models in building simulations

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Abstract Mathematical description of a linear system can be obtained in various ways. Theoretical models are desirable when basic physical laws of nature are known that describe the relationship between input and output signals, mostly in terms of differential equations. If this is not the case, approximate dynamic characterization may be received from statistical analysis of experimental data. Both of methods are jointly called system identification. The paper provides the compendium of analytical modelling techniques that are pertinent to space-load calculations. Predominantly, the conduction transfer functions and state space formulation are of interest, whereas they are more efficient than solution of finite-difference and response factors methods. If excited by meteorological conditions, the year-long simulations of the hourly heating or cooling loads for the building could be done.

1 Introduction

In order to be able to formulate mathematical description of the dynamic system one has to know the relationships between input and output signals. These can either be derived from the physical laws of nature or by using statistical evaluation of experimentally measured data. In the latter case an approximate transfer function is commonly determined based on parametrical identification of the process with least squares method. When modelling by pure identification techniques is used specific model structure shall be identified in advance, in order to define functional dependencies and prepare the design of experiment properly. The advantage of this approach is that it allows a good characterization of the building performance and minimizes computing time. On the other hand, coefficients of the mathematical formulation may vary along with measured data sets, thus do not provide a physically reasonable solution [5].

Theoretical modelling uses the lumped parameter approximation to describe the thermal behaviour of spatially distributed physical systems. This simplification can reduce the analysed system into a set of ordinary differential equations with a finite number of parameters. The conduction transfer functions and state-space formulation are of interest in the present article, since they appear to be more efficient for solving the long-term heat transfer problems than finite-difference and response factors techniques because of two reasons. As is the case with numerical methods, accuracy and stability in iterations are all functions of the time step and computing method. Secondly, compared to transfer functions all nodal temperatures must be computed at every time step even though they may not be outputs of interest [9]. Be conscious of these shortcomings, we will restrict ourselves to analytical approach that enables a more general treatment in solving the heat transfer problem in buildings.

The modelling approaches may be combined with model reduction techniques to obtain formulation of an appropriate order. To get a good description for a control system dimensioning, models of second to fourth order shall be used [11].

2 Theoretical analysis

The heat transfer problem in space-load calculations mostly centres on the solution of the heat diffusion equation, as derived by Fourier in 1822. For practical purposes, the mathematical notation can be simplified substantially providing a wall is one-dimensional and laterally isothermal [1]. As far as materials have assumed constant thermal properties, conduction through a homogeneous slab is governed by the following second-order partial differential equation

$$\frac{\partial T(x,t)}{\partial t} = \frac{\lambda}{\rho c} \cdot \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

where T is instantaneous temperature, λ, ρ and c are thermal conductivity, density and specific heat respectively. The heat flux at arbitrary wall location is thereafter described by the heat conduction law

$$\dot{q}(x,t) = -\lambda \cdot \frac{\partial T(x,t)}{\partial x} \tag{2}$$

There is no intention of deriving equations here, because it is outside the scope of this article and since many studies have been performed discussing the problem more in detail. Instead, results are only present that help us to characterize the relationship between dependent and independent variables in terms of transfer functions. A common mathematical approach to solve the differential equation is to facilitate computation by means of transform methods. Suppose the transformation into frequency domain exists just when a function is piecewise continuous with time and projection of the original converges. Laplace transforms of temperature and heat flux at inside and outside surfaces are then related by a matrix expression [6] in the form

$$\begin{bmatrix} T_i(s) \\ \dot{q}_i(s) \end{bmatrix} = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} \begin{bmatrix} T_0(s) \\ \dot{q}_0(s) \end{bmatrix} \tag{3}$$

The square matrix is called the transmission matrix with elements representing distributed thermal characteristics of the slab. To generalize this concept for a multilayer structures the wall must be decomposed into a finite number of homogeneous layers, however not necessarily equal. Treating them separately the corresponding equations are prepared for each. With components redefined in this way, the transmission matrix for a multilayer wall can be defined as a product of the matrices for various layers combined in the order in which they occur in the wall. They are all complicated transcendental hyperbolic functions [3] with a determinant equal to unity.

The response factors as well as conduction transfer functions were originally yielded using inverse transform. This procedure is very cumbersome and may lead to incorrect results [12], especially for constructions of more than two layers. Due to existing disadvantages of conventional models several new procedures have been proposed, such as state-space method where the expression is formulated by using finite-difference techniques to discretize the problem. This representation gives the relationship between state variables and the input and output vectors [2] of the system by the following equations

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{4}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \tag{5}$$

Variables are denominated according to nomenclature appeared in the literature. The exterior and interior temperature disturbances can be approaximated with a piecewise linear function.

3 The room response model

To demonstrate state-space representation of the thermal system a simplified building enclosure is considered with an envelope consisting of non-homogeneous cubic shell. The external wall has been taken as the monolithic concrete slab having constant thermal properties that incorporates a sizeable glazing area. All flanking surfaces have assumed the common materials to avoid some difficulties in the lumped model formulation so that their temperatures are represented by a single node. Even though the interfacial long-wave radiation heat exchange between enclosing surfaces can be easily taken into account [4], it was neglected for that particular case in order to guarantee the ease of this example.

Heat transfer through these building elements is assumed a one-dimensional with heat storage ability lumped in their centre. An enclosure is therefore thought as a network of first-order systems where the nodes describe the movement of thermal state points in the *n*-dimensional space. For each node, energy balances are performed [9] resulting in set of differential equations

$$\rho_i c_i V_i \frac{dT_i}{dt} = k_i \cdot S_i \left(T_j - T_i \right) \tag{6}$$

where k_i and S_i are overall heat transfer coefficient and the surface area, respectively. Following this procedure matrices in equations (4) and (5) are calculated numerically that enable straightforward conversion from the state-space representation into a transfer function. It should be understand that using this transformation some internal information about system may be lost.

4 Results and discussion

The transfer function is a mathematical representation of the relationship between the input and output of linear time-invariant systems. When the conduction heat flow through boundary walls of a building enclosure due to unsteady environmental conditions is of interest, the method stated here enables the efficient solution to this problem. As can be shown in **Figure 1**, three individual transfer functions have been obtained from the simulation. For purpose of space-load calculations

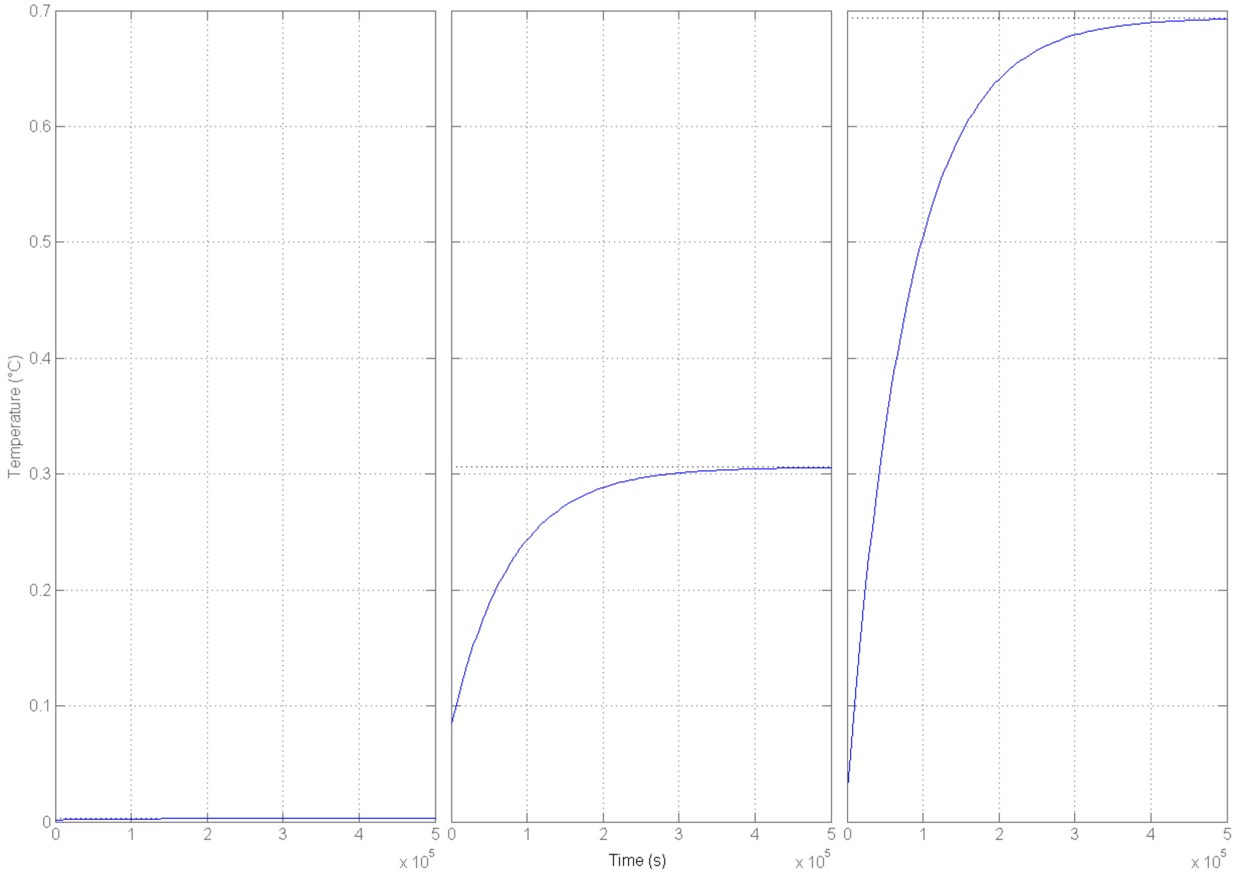

Figure 1 Step response representation from the system transfer functions

these shall be reduced into lower polynomial order by means of reduction techniques. These are essential when reducing the computational effort needed for the transfer function simulations. To get a good description of a thermal system main dynamics, models of second to fourth order are often appropriate.

5 Conclusions

The methods presented in this paper can be used for simulation of momentaneous temperature variations in an enclosure under given environmental conditions. These data are prerequisites for analysis of building long-term energy consumption, particularly if the model is exploited as part of the advanced control or building energy management system. For those cases prediction of the future system evolution must be done in advance necessarily taking the weather forecasting into account.

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