Liquid film instability model using CFD simulations

Stanislav Knotek[1], Miroslav Jícha[2]

[1] Brno University of Technology, Faculty of Mechanical Engineering, Energy Institute, Technická 2896/2, Brno, knotek@fme.vutbr.cz

[2] Brno University of Technology, Faculty of Mechanical Engineering, Energy Institute, Technická 2896/2, Brno, jicha@fme.vutbr.cz

Abstract The article presents a liquid film instability model designed using results of the set of CFD simulations. The governing equations of the model are derived using a linear equation of motion. The stability analysis is carried out by imposing a liquid surface disturbance which growth rate is investigated in dependence on the geometrical and physical configuration. The gas effect parameters, which are decisive variables in the model, are derived using results of the set of CFD simulations of turbulent flow in channel with wavy surface. The agreement between predicted and measured critical gas velocities in dependence on the liquid film thickness is very good.

1 Introduction

The liquid film sheared by gas flow in a horizontal enclosed channel has been broadly studied from half of twentieth century. From experimental studies published among others in [1], [2] and later for example in [3], it is well known that a number of different wave forms is generated on the liquid surface in dependence on the air velocity and the film thickness, eventually on the liquid flow rate. Using the measured data, so called wave regime map can be created such as is depicted in **Fig. 1** using the experimental observations of Craik published in [2].

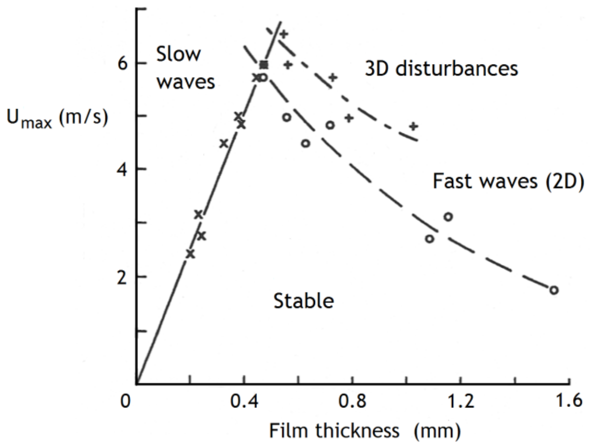

Fig. 1 Maximum air velocity plotted against thickness of water film for the three transitions. Taken from [2].

The figure shows that except the stable regime three wave types were observed. Typically, two or three dimensional waves cover the surface in dependence on the air velocity. However, for the liquid height lower than about 0.5 mm, so called slow waves appear. These long crested non-periodic waves define a third unstable regime which is generally observed for high air velocity and named solitary or capillary waves in dependence on the film thickness.

Given the various type of liquid film instabilities just mentioned, we have focused in this study only on the investigation of the conditions leading to the transition between the stable mode and two-dimensional waves on films with thickness larger than about 0.5 mm.

2 Methods

Simultaneously with the first experimental findings, the mathematical models of the initial instability as well as transitions between the wave regimes had been developed. The most used attitude is based on the linear analysis of the temporal growth of liquid surface fluctuations and the so called 'quasi-static' approximation which says that for large density and viscosity ratios between gas and liquid the wavy liquid surface can be considered as solid. Although there is uncertainty about how large these ratios must be, it is commonly assumed that the approximation is valid for air – water configuration.

2.1 Mathematical backgrounds of the linear analysis

The linear analysis of the liquid film instability is based on the Reynolds decompositions of the physical quantities into average and fluctuation parts. The liquid film displacement from its time averaged location $\bar{h}$ is defined in the form

$$h' = a\exp\{i\alpha(x - Ct)\} = a\exp(\alpha C_I t)\exp\{i\alpha(x - C_R t)\}, \quad (1)$$

where a is the amplitude of the disturbance, x is the distance in the flow direction, $\alpha = 2\pi/\lambda$ is the wavenumber defined by the wavelength λ and $C = C_R + iC_I$ is the complex wave velocity. Given the formula (1), the interfacial disturbances are assumed as harmonic waves which propagate with phase velocity C_R and grow if $C_I > 0$. The most rapidly growing disturbance is the one for which αC_I is a maximum and the so called neutral stability condition $C_I = 0$ defines the desired transition from a stable to an unstable film.

The amplitude of the wave is assumed small enough that it induced a linear response in the velocity, shear stress and pressure field in the liquid. Thus, the wall shear stress and pressure fluctuations induced by a gas flow on the liquid surface are defined by

$$\tau'_w = a\hat{\tau}_w \exp\{i\alpha(x - Ct)\}, \quad (2)$$

$$P'_w = a\hat{P}_w \exp\{i\alpha(x - Ct)\}, \quad (3)$$

where $\hat{\tau}_w$ and $\hat{P}_w$ are complex numbers. Hence, if only the real parts of (2) and (3) are considered, we get

$$\tau'_w = a\exp(\alpha C_I t)[\tau_{wR}\cos\alpha(x - C_R t) - \tau_{wI}\sin\alpha(x - C_R t)], \quad (4)$$

$$P'_w = a\exp(\alpha C_I t)[P_{wR}\cos\alpha(x - C_R t) - P_{wI}\sin\alpha(x - C_R t)]. \quad (5)$$

If the disturbances defined this way are introduced into the equations of motion, the eigenvalue problem for the wave velocity C is obtained. For more details see [4].

2.2 Governing equations

As stated Hanratty in [4], the linear stability analysis outlined in 2.1 leads for relatively thick liquid films, i.e. if $(\alpha\bar{h})(\bar{h}C_R/\nu_L)$ is a large number, and under some other restrictions into the two equation system (6)-(7) in which $\bar{u}_0$ denotes the surface velocity, g is the acceleration of gravity and ρ_L, ν_L and σ denote liquid density, viscosity and surface tension, respectively. The wave phase velocity C_R and growth rate C_I are obtained by separately equating the real and imaginary parts of (6). The resulting equations can be found in [4].

$$\begin{aligned} \alpha(\bar{u}_0 - C)\coth(\alpha\bar{h}) - \left.\frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\right|_{y=0}(\bar{u}_0 - C) \\ = G + \frac{i\hat{\tau}_w}{\rho_L}\left[\coth(\alpha\bar{h}) - \frac{1}{\alpha(\bar{u}_0 - C)}\left.\frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\right|_{y=0}\right] \\ - (\alpha\bar{h})^{1/2}\left(-i\frac{\bar{h}C}{\nu_L}\right)\alpha(\bar{u}_0 - C)^2\left[1 - \coth^2(\alpha\bar{h})\right] \\ + 4i\alpha\bar{h}\left[\frac{\bar{h}(\bar{u}_0 - C)}{\nu_L}\right]^{-1}\alpha(\bar{u}_0 - C)^2\left[\coth(\alpha\bar{h}) - \frac{1}{\alpha(\bar{u}_0 - C)}\left.\frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\right|_{y=0}\right] \end{aligned} \quad (6)$$

$$G = \frac{\hat{P}_w}{\rho_L} + \frac{\sigma\alpha^2}{\rho_L} + g \quad (7)$$

2.3 Prediction of gas effects

In order to solve the governing equation, the gas effect on the liquid surface manifested in $\hat{\tau}_w$ and $\hat{P}_w$ terms have to be substituted. Although some models of these quantities are developed in literature in dependence on channel height H, wavenumber α and gas bulk velocity U_b, we decided to use the attitude of CFD simulations.

Because of the 'quasi-static' assumption, the gas effect can be computed as the wall shear stress and pressure forces acting on the solid wavy surface induced by turbulent flow. Therefore, we have created a set of two-dimensional geometries representing a rectangular channel with flat upper wall and sinusoidal lower wall as is depicted in **Fig. 2.** The channel height was set $H/\lambda = 0.6, 0.8, 1.0, 1.2, 1{,}4$ and the wave steepness ratio λ/a was set in range from 20 to 200 while the wavelength λ has been set as a constant equal to 5 cm. The bulk velocity U_b was prescribed in range from 2 to 20 m/s.

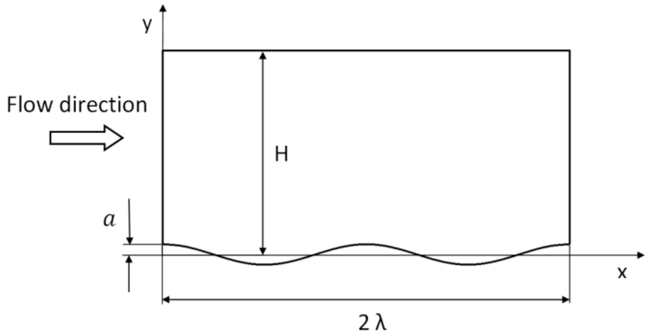

Fig. 2 Computational domain for gas effect prediction.

Using the results of the set of so defined CFD simulations computed by k-ε $V2F$ turbulence model, the gas effects have been quantified by relations (8)-(11).

$$\tau_{\mathrm{wR}} = 0.303H^{-0.219}(\lambda/a)^{-1.080}{U_b}^{1.356}/a \quad (8)$$

$$\tau_{\mathrm{wI}} = 0.126H^{-0.263}(\lambda/a)^{-0.920}{U_b}^{1.450}/a \quad (9)$$

$$\mathrm{P}_{\mathrm{wR}} = -0.815H^{-0.357}(\lambda/a)^{-0.914}{U_b}^{2.100}/a \quad (10)$$

$$\mathrm{P}_{\mathrm{wI}} = 45.936H^{-0.124}(\lambda/a)^{-1.552}{U_b}^{1.230}/a \quad (11)$$

3 Results and discussion

From the validation of gas effects models presented in previous section, it was found that however these models are accurate over broad range of conditions, they overpredict about two times the measured data for velocities lower than about 2 m/s. Because the instability threshold is expected just around this value, the separated equations resulting from (6) and (7) were implemented in Matlab using substitution of models (8)-(11) corrected by multiplier 0.5. Due to the restrictions of the governing equation mentioned in 2.2, the model is applicable only for film thickness larger than about 2 mm. The conditions under which transition occurs, i.e. $C_I = 0$, are depicted in **Fig. 3** using the dependence of the critical gas bulk velocity on the liquid film thickness. From the comparison with measurements it follows that our model prediction is very good.

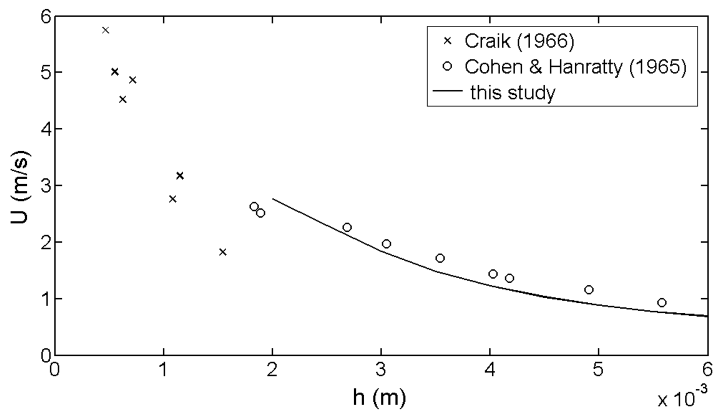

Fig. 3 Comparison of this study prediction of transition to two-dimensional waves with experimental data of Cohen & Hanratty in [1] and Craik in [2].

4 Conclusion

The liquid film stability model based on the linear analysis of momentum equations and gas effects computed using results of CFD simulations is presented. The results are very good in comparison with experimental measurements performed for films thicker than 2 mm. The instability prediction for thinner films requires another model which is the topic for further study.

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